

L. M. G. h.

JUL 13 1939

3165
57
10

TECHNICAL MEMORANDUMS
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 899

THE HYDRODYNAMIC THEORY OF DETONATION

By Heinz Langweiler

Zeitschrift für Technische Physik
Vol. 19, No. 9, 1938

FILE COPY
To be returned to
the files of the Langley
Memorial Aeronautical
Laboratory.

Washington
June 1939

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 899

THE HYDRODYNAMIC THEORY OF DETONATION*

By Heinz Langweiler

SUMMARY

The author derives equations containing only directly measurable constants for the quantities involved in the hydrodynamic theory of detonation. The stable detonation speed, D , is revealed as having the lowest possible value in the case of positive material velocity, by finding the minimum of the Du curve (u denotes the speed of the gases of combustion). A study of the conditions of energy and impulse in freely suspended detonating systems leads to the disclosure of a rarefaction front traveling at a lower speed behind the detonation front; its velocity is computed. The latent energy of the explosive passes into the steadily growing detonation zone - the region between the detonation front and the rarefaction front. The conclusions lead to a new definition of the concept of shattering power. The calculations are based on the behavior of trinitrotoluene.

INTRODUCTION

The chemical change in an explosive substance, such as the propelling charge in a firearm, generally takes place at a linear combustion speed in fractions of meters per second, whereby this rate at which the conversion zone penetrates the explosive substance is a function of the gas pressure.

Berthelot (reference 1), in 1881, discovered the peculiar fact that under suitable conditions, substantially higher conversion speeds are feasible. In his experiments with explosible gas mixtures in long tubes, he ascertained propagation speeds in the conversion zone of several kilometers per second, whereby this speed remained constant over the entire conversion distance and even remained unaffected by the initial gas pressure.

*"Beitrag zur hydrodynamischen Detonationstheorie." Zeitschrift für technische Physik, vol. 19, no. 9, 1938, pp. 271-283.

Berthelot and Dixon essayed a molecular kinetic explanation of the detonation processes and, in fact, achieved with many gases, practical values for the detonation speed.

The purely thermodynamic and aerodynamic treatment of the detonation process which disregards special molecular concepts, is attributable to Schuster (reference 2), and Chapman (reference 3), who utilized the findings obtained by Riemann (reference 4), and Hugoniot (reference 5) in the treatment of the linear compression shock in gases.

The extension of these macroscopic theories to include solid explosive substances, is due to Jouguet (reference 6) and R. Becker (references 7 and 8), the latter dealing largely with the cause of detonation stability. He proves, with the help of the thermodynamic entropy laws, the validity of Chapman's term for detonation speed. Schmidt's more recent studies of the detonation process of solid explosive substances (reference 9), containing a wealth of experimental material, are particularly well-suited to prove the validity of the hydrodynamic theory of detonation.

However, there is a gap in the orthodox theory. One obtains for the combustion gases behind the detonation front, a speed in the direction of the advancing front. The consequences of the existence of such a speed on the total system, are not considered for the reason that only the measurable quantity of this speed of the combustion gases is of influence for the detonation speed (reference 10). (Schweikert refers to this difficulty, but is of the opinion that it would void the whole hydrodynamic theory of detonation.)

To illustrate the defect of the present theory, the derivation of Chapman and Becker is briefly reviewed.

CHAPMAN AND BECKER EQUATIONS

A tube of 1 square centimeter section contains an explosive mixture under pressure p_0 , density ρ_0 , specific volume v_0 , and temperature T_0 (fig. 1).

The front of the conversion moves toward the right into the explosive at the empirically constant detonation speed D . In zone F the chemical change takes place,

the product of which, the gases of combustion, also moves toward the right at speed u .

For an observer, at rest relative to the combustion gases, hence also moving rightward at speed u , the pressure p_1 , density ρ_1 , specific volume v_1 , and temperature T_1 , prevail. Certain relations must exist between these macroscopic quantities which result from the general physical laws governing the preservation of energy, impulse, and mass.

Visualize an observer in the conversion zone F moving at detonation speed toward the right. To him the process of conversion must appear stationary if D is constant. The explosive moves at speed D into zone F from the right, while the combustion gases leave the zone at speed $D - u$ toward the left.

The preservation of the mass gives:

$$D \rho_0 = (D - u) \rho_1 \quad (1)$$

or

$$D v_1 = (D - u) v_0$$

The momentum of the explosive substance moving into the zone per second from the right is

$$B_0 = \rho_0 D D$$

The momentum of the combustion gases leaving the zone per second is

$$B_1 = \rho_1 (D - u) (D - u)$$

The difference $B_0 - B_1$ of both momentums must be equal to the effective pressure difference. Hence,

$$B_0 - B_1 = p_1 - p_0$$

and, in connection with equation (1), the momentum equation

$$\left. \begin{aligned} p_1 - p_0 &= \rho_0 D u \\ (p_1 - p_0) v_0 &= D u \end{aligned} \right\} \quad (2)$$

From the right, four different forms of energy enter the zone per second:

1. The latent chemical energy. If this amounts to U per unit mass, the leftward moving energy per second is:

$$\rho_0 D U$$

2. The kinetic energy

$$\frac{1}{2} \rho_0 D D^2$$

3. The heat contents of the undecomposed explosive gas

$$\rho_0 D c_v T_0$$

if c_v denotes the specific heat at constant volume per unit mass.

4. The external energy

$$p_0 D$$

Leaving the conversion zone per second, toward the left, are:

1. The kinetic energy of the combustion gases:

$$\frac{1}{2} \rho_1 (D - u) (D - u)^2$$

2. The heat content of the combustion gases

$$\rho_1 (D - u) c_v T_1$$

3. The external energy

$$p_1 (D - u)$$

The law of the preservation of energy therefore gives the next equation

$$\left. \begin{aligned} \rho_0 D U + \frac{1}{2} \rho_0 D D^2 + \rho_0 D c_v T_0 + p_0 D \\ = \frac{1}{2} \rho_1 (D - u) (D - u)^2 \\ + \rho_1 (D - u) c_v T_1 + p_1 (D - u) \end{aligned} \right\} \quad (3)$$

or, in conjunction with equations (1) and (2)

$$\frac{1}{2} (p_1 + p_0) (v_0 - v_1) = c_v (T_1 - T_0) - U \quad (4)$$

whereby

$$u = (v_0 - v_1) \sqrt{\frac{p_1 - p_0}{v_0 - v_1}} \quad (5)$$

and

$$D = v_0 \sqrt{\frac{p_1 - p_0}{v_0 - v_1}} \quad (6)$$

as is directly obtainable from equations (1) and (2).

Now the substitution of p_1 and v_1 or p_0 and v_0 for T_1 and T_0 in equation (4) through the phase equations, gives the relation between p_1 and v_1 , with the constants of the original gas as parameters.

This relation between p_1 and v_1 is the so-called "dynamic adiabatic" or Hugoniot curve, named after its inventor (fig. 2).

Accordingly, if in a gas of original state, p_0 and v_0 , a conversion zone travels at constant speed - saying nothing at present about the magnitude of this speed - the state of the combustion gases is represented by some point on the Hugoniot curve. According to equation (6) every point has a certain speed of propagation D and, according to equation (5), a certain speed of combustion gases.

For all points above G , there is a speed of combustion gases in the same direction as that of the propagation of the conversion zone. This conversion is associated with a marked pressure rise and passes very quickly. If the conversion of the explosive has the characteristics of this zone, it denotes detonation.

If the explosive mixture burns - i.e., if the speed of transformation is low, whereby the combustion gas pressure p_1 decreases relative to p_0 and the speed of the combustion gases is contrary to the direction of combustion - then the stage of the combustion gases is represent-

ed by the curve below E. The zone G - E cannot be practically realized.

So, even though it is true that at detonation the stage of the combustion gases, and hence the magnitude of detonation speed, is characterized by a point on the Hugoniot curve above G, this still does not determine which point defines the actually occurring detonation speed.

The only stable point on the curve, according to Chapman and Becker, is the contact point J of the tangent from A.

For angle φ of the tangent, it is

$$\tan \varphi = \frac{p_1 - p_0}{v_0 - v_1}$$

or, according to equation (6),

$$D = v_0 \sqrt{\tan \varphi} \quad (7)$$

Since $\tan \varphi$ is computable from the dynamic adiabatic, expressions for D and u containing only macroscopic, thermodynamic equations can be adduced.

These are:

$$D = \frac{k+1}{k} \sqrt{k \frac{R}{M} T_1} \quad (8)$$

$$u = \sqrt{\frac{1}{k} \frac{R}{M} T_1} \quad (9)$$

The constants

k, ratio of specific heat

R, gas constant

M, molecular weight

apply to the temperature T_1 of the combustion gases.

Forming the difference $a = D - u$ gives

$$a = \sqrt{k \frac{R T_1}{M}} \quad (10)$$

But this expression presents the magnitude of the velocity of sound in the combustion gases.

Accordingly, it may be stated: The detonation speed is given through the sum of the speed of combustion gases and sonic velocity in the combustion gases.

The detonation temperature necessary for computing D is obtained from the specific heat and the heat content of the explosive whereby the temperature rise due to the density increase on passage through the wave front is still accounted for through the dynamic adiabatic curve. The detonation speeds computed this way are in good agreement with practical experience, hence affording conclusions regarding the fundamental proof of the concept.

The hydrodynamic derivation of formulas for solid and liquid explosives follows the same method, save that the specific volume of the gas molecules must be allowed for, as is customary in internal ballistics, by the introduction of Abel's equation, a modification of van der Waal's form.

Quantitative calculation of the detonation speed of solid explosives being out of the question, on account of ignorance of the covolume of gases at high pressures and temperatures, the measured detonation speed is used instead for calculating the covolumes.

In any case, the derived expressions indicate an experimentally confirmed relationship with density ρ_0 , which does not obtain for the gas formulas (8) and (9).

DERIVATION OF NEW EXPRESSIONS

The first task is the derivation of expressions for detonation speed D , and material speed u , which are free from the concept of detonation temperature and contain solely constants directly amenable to measurement. The derivation is to be carried out direct for solid and liquid explosives. By disregarding the covolume, the equations become valid for gaseous explosives.

In the following, various concepts of internal ballistics are employed.*

The combustion of a quantity of explosive of weight L in a closed bomb of constant volume V creates a terminal pressure p_E , called explosion pressure.

For the covolume of gases per unit weight α , Abel's equation gives

$$p_E = f \frac{L}{V - \alpha L}$$

or, for the specific weight of the gases $\frac{L}{V} = \rho_0$,

$$p_E = f \frac{\rho_0}{1 - \alpha \rho_0} \quad (11)$$

f and α can be determined from pressure measurements in the bomb by means of equation (11).

The phase equation

$$p_E (1 - \alpha \rho_0) = \frac{R T_E}{M} \rho_0$$

discloses that the experimentally defined constant f has the value

$$f = \frac{R T_E}{M} \quad (12)$$

with T_E , explosion temperature. T_E is the temperature of the explosion gases existing when combustion takes place without external energy input; i.e., for instance, in the closed bomb.

The gas under explosion pressure is to be adiabatically released up to zero pressure by means of a plunger. With k as exponent of the adiabatic release, the energy performed on the piston is

*H. Langweiler, Z.f.d. ges. Schiess- u. Sprengstoffwesen: Development and experimental check of closed mathematical expressions for internal ballistic constants, particularly for the maximum gas pressure and the muzzle velocity. (To be published.)

$$A = \frac{p_E (V - \alpha L)}{k - 1} \quad (13)$$

which follows directly by integration of the adiabatic curve, according to Poisson. The energy capacity A of the gas in the form of equation (13) is called the potential gas energy. But this energy A must be equal to the latent energy contained in the quantity of explosive L before combustion. If this amount per unit weight is U , we have

$$A = \frac{p_E (V - \alpha L)}{k - 1} = L U$$

or, with observance of equation (11):

$$f = (k - 1) U$$

$$k = 1 + \frac{f}{U} \quad (14)$$

Constants f and U each have the dimension of an energy (m kg) per unit weight (kg), hence the dimension of a length (m). The exponent k is an average value; but differs very little from that (≈ 1 percent) computed from the specific heat, as will be seen from the example later on.

Other than density ρ_0 of the explosive and covolume α_1 of the detonation gases, the expression for the detonation speed is to contain only the measurable constants U and f .

Figure 3 is another sketch of the detonation process. For the sake of clearness the two sides of the conversion zone carry pistons which at pressure p_0 and p_1 act on the static explosive or the moving combustion gases. The conception of such pressure forces is necessary as otherwise no stationary phase would be conceivable.

As the conversion zone moves toward the right for a distance D , the combustion gases move for a distance u , in the direction of the advancing zone. Since the pressure p_1 is to be constant within the combustion gases, the piston A likewise must travel a distance u , toward the right.

The mass, impulse, and energy equations for an observer moving with the detonation front, can be written in known manner. The pressure p_0 being very small relative to the detonation pressure p_1 ($p_0 = 0$ for solid explosives), its effect can be disregarded.

Mass:

$$D \rho_0 = (D - u) \rho_1 \quad (15)$$

Impulse:

$$p_1 = \frac{\rho_0}{g} D u \quad (16)$$

(g is acceleration of gravity)

Energy:

$$\left. \begin{aligned} D \rho_0 U + \frac{1}{2g} \rho_0 D D^2 \\ = \frac{p_1 (D-u) (1-\alpha_1 \rho_1)}{k-1} \\ + \frac{1}{2g} \rho_1 (D-u) (D-u)^2 + p_1 (D-u) \end{aligned} \right\} \quad (17)$$

The left-hand side of the last equation gives the latent and kinetic energy of the explosive movement per second into the zone from the right.

Behind zone F, leftward, a gas space of magnitude $D - u$ per second is created, wherein the pressure p_1 prevails because the piston A, as seen from the detonation front, moves a distance $D - u$ to the left per second. Now the potential gas energy of this newly created space is (similar to equation (13)):

$$\frac{p_1 (D - u) (1 - \alpha_1 \rho_1)}{k - 1}$$

The factor $(1 - \alpha_1 \rho_1)$ accounts for the reduction in gas space due to the covolume of the gases.

The total gas space moves at a speed $(D - u)$ toward

the left. Its additional kinetic energy, accordingly, is:

$$\frac{1}{2g} \rho_1 (D - u) (D - u)^2$$

As seen by the observer, the work $p_1 (D - u)$ is performed on the plunger. The three forms of energy appearing per second on the left side of the detonation front must, together, equal the energy per second entering from the right. The result is equation (17).

Quantities p_1 and ρ_1 may be substituted herein through (15) and (16). With consideration of $U (k - 1) = f$, we have:

$$(k - 1) D^2 + f g = \frac{k - 1}{2} (D - u)^2 + (D - u) u \left(1 - \rho_0 \frac{D}{D - u} \alpha_1 \right) + (k - 1) u (D - u)$$

and through direct calculation

$$D = \frac{1}{1 - \alpha_1 \rho_0} \left(\frac{f g}{u} + \frac{k + 1}{2} u \right) \quad (18)$$

or abbreviated:

$$\eta = 1 - \alpha_1 \rho_0 \quad (19)$$

For negligible covolume $\eta = 1$. In

$$D = \frac{1}{\eta} \left(\frac{f g}{u} + \frac{k + 1}{2} u \right) \quad (20)$$

the detonation speed D is shown as function of the sole variable u , namely, the material speed.

Since f has the dimension of length and g is the acceleration, the validity of the dimension is proved.

Figure 4 illustrates the function (20). The existence of a minimum is immediately apparent. The correlated pair of values is obtained by differentiation of (20):

$$\frac{dD}{du} = 0 = \frac{1}{\eta} \left(-\frac{f g}{u^2} + \frac{k+1}{2} \right)$$

Hence

$$u = \sqrt{\frac{2}{k+1} f g} \quad (21)$$

and, with (20)

$$D = \frac{1}{\eta} \sqrt{2(k+1) f g} \quad (22)$$

or

$$D = \frac{1}{\eta} (k+1) u \quad (23)$$

Equations (21) and (22) are equivalent to Chapman's equations (8) and (9) as closer examination shows. Aside from the effect η of the covolume, they merely contain the constant f known from bomb tests and the latent energy U , because k itself is $= 1 + f/U$.

From the agreement of the value of the lowest possible detonation speed, according to (20), with that derived as stable from the Hugoniot curve, the following conclusion can be drawn:

The constant detonation speed ultimately attained in a detonation process, is the lowest conversion speed possible, according to the laws of physics, provided that a sufficiently strong initial ignition creates a combustion-gas speed in the direction of the advancing zone of transformation.

The magnitude of the sonic velocity in the combustion gases is determined as follows: If E is the modulus of elasticity of the medium, the sonic velocity a in any medium follows the law:

$$a = \sqrt{\frac{E}{\rho}}$$

The modulus of elasticity for ideal gases is $E = k p$ ob-

tained by differentiation of the adiabatic curve

$$p \left(\frac{1}{\rho} \right)^k = \text{constant}$$

But with

$$p_1 \left(\frac{1}{\rho_1} - \alpha_1 \right)^k = \text{constant}$$

as adiabatic curve of the combustion gases, its modulus of elasticity becomes:

$$E = \frac{p_1 k}{1 - \alpha_1 \rho_1}$$

Accordingly, the velocity of sound in these gases is

$$a = \sqrt{\frac{k p_1 g}{\rho_1 (1 - \alpha_1 \rho_1)}} \quad (24)$$

and, after replacing p_1 and ρ_1 by equations (15) and (16)

$$a = \sqrt{\frac{k u (D - u)}{1 - \alpha_1 \rho_0 \frac{D}{D - u}}}$$

and, with the abbreviation $\eta = 1 - \alpha_1 \rho_0$:

$$a = \frac{1}{\eta} (k + 1) u - u \quad (25)$$

On comparison with equation (23), it is seen that the velocity of sound is equal to the difference between detonation speed D and combustion-gas speed u , that is:

$$\boxed{a = D - u} \quad (26)$$

The result is in accord with Chapman and Becker's findings.

At sonic velocity itself, the equation is:

$$a = \left(\frac{k+1}{\eta} - 1 \right) u$$

or

$$a = \left(\frac{k+1}{\eta} - 1 \right) \sqrt{\frac{2}{k+1} f g} \quad (27)$$

The detonation pressure p_1 can be computed according to equation (16):

$$p_1 = \frac{\rho_0}{g} D u$$

The introduction of the values (21) and (22) gives:

$$p_1 = \frac{1}{\eta} 2 f \rho_0 \quad (28)$$

Expressing the gas pressure of the explosives resulting in the original volume under the slow combustion of the explosive, again as explosion pressure p_E , and the covolume for this pressure with α_E , Abel's equation

$$p_E = \frac{f \rho_0}{1 - \alpha_E \rho_0}$$

is applicable.

From equation (28) then follows:

$$p_1 = 2 p_E \frac{1 - \alpha_E \rho_0}{1 - \alpha_1 \rho_0} \quad (29)$$

and with it, the important result that the detonation pressure is twice as great as the explosion pressure, disregarding for the time being the effect of the difference in covolume. For gases, it is exactly:

$$p_1 = 2 p_E \quad (30)$$

For the combustion-gas density, it is, according to equation (15), after insertion of the D and u value:

$$\rho_1 = \frac{k+1}{k+1-\eta} \rho_0 \quad (31)$$

The detonation temperature T_1 can be computed in the following manner. The combustion gases follow the phase equation:

$$p_1 \left(\frac{1}{\rho_1} - \alpha_1 \right) = \frac{R}{M} T_1$$

But, according to equation (12), it is:

$$f = \frac{R}{M} T_E$$

and consequently,

$$\frac{R}{M} = \frac{f}{T_E}$$

Hence,

$$T_1 = \frac{p_1 \left(\frac{1}{\rho_1} - \alpha_1 \right)}{f} T_E \quad (32)$$

p_1 and ρ_1 may be expressed with D and u :

$$T_1 = \frac{\rho_0 D u}{f g} \left(\frac{D-u}{\rho_0 D} - \alpha_1 \right) T_E$$

Substituting equations (21) and (22) for D and u , we have the simple relation:

$$T_1 = \frac{2k}{k+1} T_E \quad (33)$$

The detonation temperature T_1 is, independent of the covolume of the combustion gases, greater by the factor $\frac{2k}{k+1}$ than the explosion temperature T_E of the explosive.

The aim of the first problem, the derivation of the quantities incident to the detonation of any explosive, containing - besides covolume - only the constants f and U , is herewith reached. Obviously, equation (22) is applicable for the determination of covolume α_1 of the combustion gases, in case the detonation speed D is measured.

THEORETICAL PROOF OF THE EXISTENCE OF AN INCREASING DETONATION HEAD

The second problem is the study of the consequences of a stationary detonation process illustrated in figure 3. The assumption of a plunger A acting externally at magnitude of detonation pressure p_1 and moving at combustion-gas speed u in direction of the moving detonation front is, as already stated, necessary for the explanation of the stationary processes in the transformation zone F .

At the initiation of a detonation process, the plunger A is represented by the nascent gases of the initiator. The energy of these gases, however, would only suffice to drive the detonation front a very short way into the explosive. Once the action of the initiator gases has stopped, the cause for the positive speed of the combustion gases must naturally cease and the detonation come to an end.

Riemann stated that a shock wave in gases could be maintained permanently only through external forces. But since experiments indicate that a detonation, once initiated, travels with great constancy over any long distance, the explanation of the detonation process is in need of a supplement. This necessity becomes especially evident in a simple impulse consideration comprising a whole detonating system.

Visualize a long column of explosive freely suspended horizontally. Ignition is to take place at one end. After the detonation wave has reached the other end, the then-formed gas of pressure p_1 and density ρ_1 would have to move as a whole at material velocity u in the direction of the detonation front. Such a motion is, of course, impossible, because no external forces which could cause a total impulse, act on the freely floating explosive.

The effect of the initial charge can be disregarded, since the column may be of any length.

An energy balance comprising the whole system also shows the incompleteness of the initially employed presentation; for the total energy of the combustion gases (potential and kinetic) of a freely suspended detonating column of explosive is, according to calculation, greater than the latent energy originally existing in the explosive.

In the calculation this difference is covered by the external energy of the plunger, but in the absence of external energies, as is the case for a column of explosive left to itself, no constant detonation speed would probably exist.

The difficulties introduced with the impulse and energy balance of explosive systems left to themselves, must be voided by assuming a second wave front in the combustion gases. The speed w of this front relative to the quiescent explosive, must be greater than the speed of the gases u , but smaller than the detonation speed. On passing through this front the detonation pressure itself must drop to a value below that of the explosion pressure.

Figure 5 illustrates the process of detonation of an explosive substance on the assumption of a rarefaction front advancing at lower speed behind the detonation front. This rarefaction front moves at subsonic velocity relative to the combustion gases; that is, it is debatable, while a rarefaction shock of supersonic velocity would be impossible.

A - B is the explosive column made to detonate. Ignition takes place at A, where a solid abutment rests relative to the original explosive and which is therefore unable, in contrast to a moving plunger, to perform work on the total system.

The significance of this "damming up" is referred to elsewhere. For the present, it may be stated that the omission of A modifies the speed of the rarefaction front but not the detonation speed.

F_1 is the detonation front wherein the pressure rises from p_0 to the value p_1 . F_2 indicates the rarefaction front, within which p_1 drops to p_2 and the ma-

material velocity changes from value u to zero. Nothing is said about the form of the rarefaction front other than assuming that its extent is small compared to the distance $F_1 F_2$. The region between the two fronts is called detonation head, and the distance $F_1 F_2$, the head length.

Figure 5 indicates how, as a result of the difference in speed of the fronts, the length of the head continues to grow with time and, hence, with the detonation distance. Since the abutment A is fixed, the gas behind the rarefaction front F_2 must be at rest; i.e., its material velocity must become zero in case the process in the presented form is to be stationary. Suppose the values for this new stage of the combustion gases are: p_2 , ρ_2 , α_2 .

At present the combustion-gas speed u is confined within the head $F_1 F_2$. Before front F_1 and behind front F_2 the material is quiescent. As the head becomes longer in unit time the momentum of the gases within the fronts increases with respect to time. The cause for this growth in momentum lies in the pressure difference $p_2 - p_0$ existing at either side of the head. The pressure p_2 must be lower than the explosion pressure p_E , for the energy increment of the detonation head is necessarily at the expense of combustion-gas energy of stage 2.

Assuming the nonexistence of the abutment at A , the inertia resistance of the gases in proximity of A is to be substituted. Then the gases behind the rarefaction front are subject to leftward acceleration - i.e., opposite to the gas speed within the head - so that the impulse law is always complied with. Such leftward motion of the combustion gases in stage 2 is followed by a drop in pressure p_2 relative to the pressure with fixed abutment.

But a drop in p_2 calls for a decrease in temporal rise of gas momentum within the head; i.e., the head length would increase less rapidly in time or, ultimately, the speed of the rarefaction front would more closely approach the speed of the detonation front. This fact is of importance for the effect of the explosive, as will be shown later on.

In the following, the quantitative calculation of the speed of the rarefaction front with fixed abutment A is carried out. It could also be made for absent abutment,

although not without difficulties because of the nonuniform processes. Since, as ascertained qualitatively, the existence of material speed behind the detonation front is possible by the energy law as well as the impulse law in case of an extension of the concept regarding the further destiny of the combustion gases, equations (21) and (22), established for detonation speed and material speed, will be considered applicable.

Assume that figures 5c and 5d are instantaneous photographs of the detonation process with a time interval of 1 second. While within this interval point H of the detonation front has reached by quantity D the point G, the point C of the rarefaction front has traveled the distance w to point K. Consequently, the difference of the head lengths CH and KG after 1 second, is $D - w$. The increment of the mass of the head is

$$\frac{\rho_1}{g} (D - w)$$

As this mass moves at material speed u, the rise in momentum of the head per second is

$$\frac{\rho_1}{g} (D - w) u$$

This rise must equal the effective pressure difference at both ends of the head. This gives the impulse equation ($p_0 \ll p_2$):

$$p_2 = \frac{\rho_1}{g} (D - w) u \quad (34)$$

Each second the latent energy $\rho_0 D U$ is transformed into potential and kinetic gas energy of the combustion gases. New energy forms per second, are: potential gas energy in stage 2, of the order of

$$\frac{p_2 w (1 - \alpha_2 \rho_2)}{k - 1}$$

the potential gas energy at stage 1, of the order of

$$\frac{p_1 (D - w) (1 - \alpha_1 \rho_1)}{k - 1}$$

and the kinetic gas energy at stage 1, of the value of

$$\frac{1}{2g} \rho_1 (D - w) u^2$$

Hence,

$$\begin{aligned} \rho_0 D U = & \frac{p_1 (D - w) (1 - \alpha_1 \rho_1)}{k - 1} \\ & + \frac{p_2 w (1 - \alpha_2 \rho_2)}{k - 1} + \frac{1}{2g} \rho_1 (D - w) u^2 \end{aligned}$$

or (equation 14)

$$\begin{aligned} \rho_0 D f = & p_1 (D - w) (1 - \alpha_1 \rho_1) \\ & + p_2 w (1 - \alpha_2 \rho_2) \\ & + \frac{k - 1}{2g} \rho_1 (D - w) u^2 \end{aligned} \quad (35)$$

The law of preservation of mass for an observer sitting in the rarefaction front gives:

$$\rho_2 w = \rho_1 (w - u) \quad (36)$$

The solution of the variables w , p_2 , α_2 , and ρ_2 in equations (34), (35), and (36) is contingent upon α_2 .

In the detonation of gaseous explosives, the covolumes α_1 and α_2 are negligible, hence w can be computed. For gases, it is:

$$w = K(k) D \quad (37)$$

whereby $K(k)$ is a function of k only:

$$K(k) = \frac{3 - k + \sqrt{(3k - 1)^2 + 8k(k + 1)}}{4(k + 1)}$$

Putting k roughly at 1.2, we have:

$$K(k) = 0.8 \quad \text{and} \quad w = 0.8 D$$

The speed of the rarefaction front in gases is thus only about 20 percent lower than the detonation speed or, in other words, the head length amounts to about 20 percent of the distance covered by the detonation front.

For solid or liquid explosives whose covolumes α_1 and α_2 are not negligible, w can be computed in the following manner. As shown by A. Schmidt (reference 9), the covolume α_1 of the combustion gases, is a function of the gas density ρ_1 and, consequently, of the original density ρ_0 of the explosive. He illustrates this relationship as following from detonation-speed measurements for different explosive densities with curves and arrives at a decrease in covolume for increasing gas density, which seems extremely likely.

Now the covolume is not only dependent on the density but, with constant density, on the temperature as well.

Visualize a molecule heap of great density but low temperature. Then the individual molecules penetrate into their common action spheres merely as a result of their need for space (deformation of "molecule envelopes"). If a rise in density is accompanied by constant temperature, the depth of penetration is greater; that is, the covolume becomes less. If, with constant density, the temperature is increased, the molecules - as a result of their then-increased kinetic energy - penetrate still deeper into the envelopes of their neighbors with a further reduction in covolume as a result.

Hence, the factor best suited to describe, as sole independent variable, the behavior of the covolume of a gas, is the pressure because it is, according to the phase equation, proportional to the density as to the temperature.

The relation $\alpha = \alpha(p)$ obtained, say, for a certain gas, may therefore suffice to define the magnitude of α when p alone is known. This functional relationship between the pressure and covolume of a certain explosive

$$\alpha = \alpha(p) \quad (38)$$

can also be established from detonation-speed measurements

for different explosive densities. The covolume α_1 is solved according to equation (22), and the correlated pressure p_1 , according to equation (28). Several pairs then give the desired curve $\alpha = \alpha(p)$.

Following this preparatory step, the speed of the rarefaction front is calculable, since the magnitude of α_2 is now defined by the existing pressure p_2 . The four available equations, (34), (35), (36), and (38) contain the four variables w , p_2 , α_2 , and ρ_2 . An explicit calculation of w would give an expression too complex to handle. Much quicker results are obtained when computing ρ_2 according to (36), and p_2 according to (34) for an assumed w . The $\alpha = \alpha(p)$ curve gives the correlated α_2 value for p_2 . These four variables must satisfy equation (35). The process is repeated.

The temperature T_2 behind the rarefaction front follows from the phase equation at

$$T_2 = \frac{p_2 \left(\frac{1}{\rho_2} - \alpha_2 \right)}{f} T_E \quad (39)$$

The quantities in the detonation of trinitrotoluene of density 1.59 kg/liter are now computed by means of the developed equations, on the basis of Friedrich's test data of detonation speeds at different densities quoted by A. Schmidt.

TABLE I

Detonation speed of trinitrotoluene in relation to density according to Friedrich

Density ρ_0 (kg/liter)	Detonation speed D (m/s)
1.0	4,700
1.29	5,900
1.46	6,500
1.59	6,900

The heat content is:

$$U = 1,085 \text{ (cal/kg)}$$

$$= 4.63 \times 10^5 \text{ (m kg/kg) or (m)}$$

According to H. Kast (reference 11), the powder constant f of trinitrotoluene is

$$f = 8.08 \times 10^4 \text{ (m)}$$

hence the exponent of adiabatic expansion, according to equation (14):

$$k = 1 + \frac{f}{U} = 1.175$$

(For U the total heat volume, strictly speaking, should be set up to absolute zero point; but the error caused by referring the recorded heat content to 0°C . may be ignored.)

Equation (21) gives the speed of the combustion gases ($g = 9.81 \text{ (m/s}^2\text{)}$)

$$u = 855 \text{ (m/s)}$$

The factor $\frac{1}{n}$ or the covolume α_1 is obtainable from equation (23) for the different detonation speeds, while the detonation pressure follows from equation (16).

Table II contains the values thus obtained from Friedrichs' measurements.

TABLE II

Covolume α_1 and detonation pressure p_1 of trinitrotoluene according to the developed equations

Detonation speed $D \text{ (m/s)}$	Covolume $\alpha_1 \text{ (liters/kg)}$	Detonation pressure $p \text{ (kg/cm}^2\text{)}$
4,700	0.60	4.09×10^4
5,900	.53	6.62×10^4
6,500	.49	8.27×10^4
6,900	.46	9.55×10^4

The association between covolume and pressure $\alpha = \alpha(p)$, taken from this table, is graphically shown in figure 6.

The explosion pressure is, according to (11)

$$p_E = 7.18 \times 10^4 \text{ (kg/cm}^2\text{)}$$

Since its solution is contingent upon the knowledge of α_E , the calculation must proceed on the basis of the now-known function $\alpha = \alpha(p)$. The correlated covolume is

$$\alpha_E = 0.517 \text{ (liter/kg)}$$

The detonation pressure of trinitrotoluene of density 1.59 kg/liter is thus about 30 percent higher than the explosion pressure which, for gaseous explosives, would amount to 100 percent. The density of the detonation gases behind the transformation zone follows from equation (31) at

$$\rho_1 = 1.82 \text{ (kg/liter)}$$

and the explosion temperature from equation (12) at

$$T_E = \frac{fM}{R}$$

or, since the molecular weight $M = 32.72$,

$$T_E = 3,120^\circ \text{ K.}; \quad T_E = 2,847^\circ \text{ C.}$$

The detonation temperature is, according to (33):

$$T_1 = 1.08 T_E; \quad T_1 = 3,380^\circ \text{ K.}; \quad T_1 = 3,107^\circ \text{ C.}$$

These quantities are compared with Schmidt's findings.

Except for T_1 the agreement is good. The values for the phase quantities behind the rarefaction front are obtained by approximation.

Table IV gives the quantities involved in the detonation, along with the explosion quantities for comparison.

TABLE III

Comparison of computed detonation quantities of trinitrotoluene of 1.59 kg/liter density and 6,900 m/s detonation speed

Detonation quantity	Calculated according to		
	Author's equations	Chapman's equations	
k	1.175	1.16	-
u	855	891	(m/s)
α_1	0.46	0.45	(liters/kg)
p_1	9.55×10^4	9.65×10^4	(kg/cm ²)
ρ_1	1.82	1.83	(kg/liter)
T_1	3,107	3,630	(°C.)

TABLE IV

Quantities involved in the detonation of trinitrotoluene of 1.59 kg/liter density

Detonation quantities	Notation	Value
Density of solid explosive	ρ_0	1.59 (kg/liter)
Detonation speed	D	6,900 (m/s)
Explosion pressure	p_E	7.18×10^4 (kg/cm ²)
Covolume of explosion pressure	α_E	0.517 (liter/kg)
Explosion temperature	T_E	2,847 (°C.)
Exponent of adiabatic expansion	k	1.175
Material speed of combustion gases behind detonation front	u	855 (m/s)
Detonation pressure	p_1	9.55×10^4 (kg/cm ²)
Covolume behind detonation front	α_1	0.46 (liter/kg)
Combustion-gas density behind detonation front	ρ_1	1.82 (kg/liter)
Detonation temperature	T_1	3,107 (°C.)
Speed of rarefaction front	w	3,720 (m/s)
Pressure behind rarefaction front	p_2	5.03×10^4 (kg/cm ²)

(Continued on p. 26.)

TABLE IV (Cont.)

Quantities involved in the detonation of trinitrotoluene
of 1.59 kg/liter density

Detonation quantities	Notation	Value
Covolume behind rarefaction front	α_2	0.58 (liter/kg)
Gas density behind rarefaction front	ρ_2	1.40 (kg/liter)
Temperature of gases behind rarefaction front	T_2	2,350 ($^{\circ}\text{C.}$)

The obtained values are illustrated in figure 7. A striking feature is the great distance of the two fronts, amounting to almost 50 percent of the travel of the detonation front as compared to about 20 percent for gaseous explosives.

According to figure 7c, the material speed u prevails only between the two fronts, but is zero in the quiescent explosive and during the terminal stage of the combustion gases. Figure 7d gives the energy density along the detonation path, with allowance for the potential as well as the kinetic gas energy. The shifting of the energy into the detonation head is plainly seen. The integral of this energy distribution must naturally equal the total latent energy of the explosive released by the detonation front F_1 . Figure 7e illustrates the effect of the mass. Here also the integral of the mass distribution must agree with the total mass of explosive engendered by the detonation.

Table V gives the individual energies for a column of explosive of 1 cm^2 section and D length at detonation.

The sum of the total energy of the detonation head and of the potential energy of the residual combustion gases behind the rarefaction front is numerically equal to the latent energy. Consequently, the total energy of the head is 60 percent of the latent energy of the explosive involved in the detonation. Thereby the kinetic energy within the head is considerably smaller than the potential.

TABLE V

Distribution of energy in a detonating trinitrotoluene column of density 1.59 kg/liter and length D

Energy form	Magnitude (m kg)
Latent energy	5.08×10^8
Potential energy of detonation head	2.83×10^8
Kinetic energy of detonation head	$.22 \times 10^8$
Total energy of detonation head	3.05×10^8
Potential energy of gases behind rarefaction front	2.02×10^8

DEDUCTIONS FOR THE CONCEPT OF SHATTERING POWER

Following proof of the existence of a detonation head of increasing length in time as sequence of the physical fundamental laws, the question of shattering power of an explosive is discussed.

Visualize at B (fig. 5) a steel plate hit vertically by the detonation front F_1 .

R. Becker holds the detonation pressure p_1 responsible for the effect on the steel plate. But a pressure alone cannot be decisive for an effect. Assume a closed bomb of very small volume under high pressure p_1 . Owing to the pressure, the walls expand and the pressure drops immediately to a fraction of the initial value, since the small energy storage is not able to maintain the initial pressure permanently.

While the pressure alone is decisive for the effect on absolutely rigid walls, the magnitude of all effects exerted on deformable bodies (i.e., all actually existing bodies) is contingent upon the available energy.

Take a closed bomb of large content filled with a gas, the pressure of which is lower than the tensile strength of the bomb walls, the potential gas energy being suffi-

cient to withstand the work of deformation. Even then the effect obviously will be zero, on account of the low pressure.

The shattering effect therefore does not increase with the available energy only but also with the height of the pressure.

The basis of our concept of detonation process now suggests consideration of the total energy of the detonation head as energy reservoir for a shattering effect, while the effective pressure is equal to the detonation pressure p_1 . (The increase in hitting pressure due to the combustion-gas momentum is negligible.)

But the head energy is a function of the amount or volume of detonating explosive. Denoting this energy of a detonating explosive of 1 cm section and 1 m length with E_K , we find for the cited trinitrotoluene

$$E_K = 4.22 \times 10^4 \left(\frac{\text{m kg}}{\text{m cm}^2} \right)$$

However, this head energy E_K of a certain explosive volume cannot as yet be a fitting term for the shattering power, because the detonation pressure itself must, as qualitatively deduced, be present. The higher the detonation pressure, the higher the detonation speed and the smaller the time interval necessary to course through a 1-meter column. If this time is $t = \frac{1}{D}$ (s), the shattering power B of an explosive appears to be adequately expressed by

$$B = \frac{E_K}{t} = E_K D \left(\frac{\text{m kg}}{\text{m cm}^2 \text{ s}} \right)$$

which presents the shattering power by an energy per volume per time; i.e., performance per unit volume. Substituting the kilowatt and liter for m kg/s and $\text{cm}^3 \text{ m}$, respectively, we have

$$B = E_K D \text{ (kw/liter)} \quad (40)$$

Thereby the head energy E_K for an explosive of 1-meter length and 1-liter volume, i.e., 10 cm^2 section, is computed in the described manner.

The shattering power could equally be referred to the unit weight of the explosive but since in practice a certain volume is available for an explosive, the unit volume is more appropriate.

For trinitrotoluene of 1.59 kg/liter density, it gives

$$B = 2.98 \times 10^7 \text{ (kw/liter)}$$

It is readily seen that the shattering effect of an explosive is essentially contingent upon the detonating quantity rather than upon the data of detonation pressure only, which is not dependent on the quantity of material.

In the absence of abutment A (fig. 5), the gases in stage 2 move under pressure decrease, opposite to that of the rarefaction front. The rarefaction advances faster as a result of this pressure decrease.

After a certain detonation path, the distance of both fronts therefore is smaller without than with A. Hence, since the head energy, despite the constant detonation pressure, is dependent on A, it is readily apparent why the effects of explosives in the dammed-up stage are greater than those in the free stage.

CONCLUSION

In conclusion, we shall analyze the detonation mechanism. The accord of the detonation speed derived from the minimum condition with practical experience suggests the assumption that the magnitude of detonation speed is actually dependent upon the macroscopic quantities only, such as heat content, explosion pressure, etc., whereas the physical state of the individual explosive particles is unimportant, as exemplified by the quality of detonation speed of solid and liquid explosives of the same density.

The reaction process, which in explosions and combustions defines the linear speed of transformation, plays no part at all in the phenomenon of detonation.

It may therefore be assumed that the explosive before the detonation front is at a stage independent of its initial physical state. The author, in consequence, assumes

that the radiation of the gases of combustion is sufficient to activate a thin layer of the explosive before the detonation front and here create the explosion pressure p_E and the explosion temperature T_E . (Then the magnitude of the detonation speed is given only by the propagation speed of a shock wave in a gas of pressure p_E and temperature T_E .) (That this speed of propagation is greater than the normal velocity of sound in this gas, is the result of the high amplitude of the shock.)

A molecular-kinetic analysis of the detonation phenomenon (reference 12) can be made backward from the already available thermodynamic data precisely as the nature of the velocity of sound on the basis of the known thermodynamic formula was recognized as the quantity of the molecular speed of the particular gas multiplied by a certain constant.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCES

1. Bollé, E.: Handbuch d. physikal. u. techn. Mechanik von F. Auerbach u. W. Hort, vol. 6; ch. on Explosion und Explosionswellen. Leipzig, 1928.
2. Schuster, A.: Phil. Trans., vol. 184A, 1893, p. 152.
3. Chapman, D. L.: Phil. Mag., vol. 47, 1899, p. 90.
4. Riemann, B.: Göttinger Nachr., vol. 8, 1860, p. 156.
5. Hugoniot, H.: Jour. d. l'école polytechn. Paris, 1887 and 1889.
6. Jouguet, E.: Mécanique des Explosifs. Paris, 1917.
7. Becker, R.: Z. f. Phys., vol. 8, 1922, pp. 321-362.
8. Becker, R.: Physikalisches über feste und gasförmige Sprengstoffe. Z. f. techn. Phys., vol. 3, 1922, pp. 152 and 249.
9. Schmidt, A.: Über die Detonation von Sprengstoffen und die Beziehung zwischen Dichte und Detonationsgeschwindigkeit. Z. f. d. ges. Schiess- u. Sprengstoffw., vol. 30, 1935, pp. 364-369; vol. 31, 1936, pp. 8-13, 37-42, 80-84, 114-118, 149-153, 183-187, 218-222, 248-252, 284-288, 322-327.
10. Schweikert, G.: Z. f. d. ges. Schiess- u. Sprengstoffw., vol. 32, 1937, p. 88.
11. Granz, C.: Lehrbuch d. Ballistik, vol. II, p. 107. Berlin, 1926.
12. Finkelburg, W.: Zur Theorie der Detonationsvorgänge. Z. f. d. ges. Schiess- u. Sprengstoffw., vol. 31, 1936, pp. 109-113.

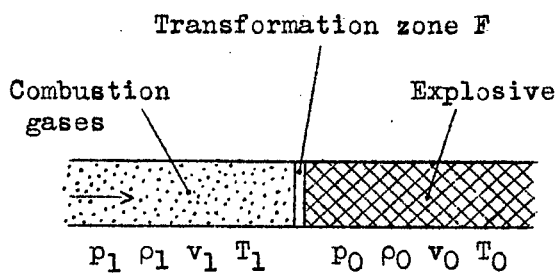


Figure 1.- Detonation process.

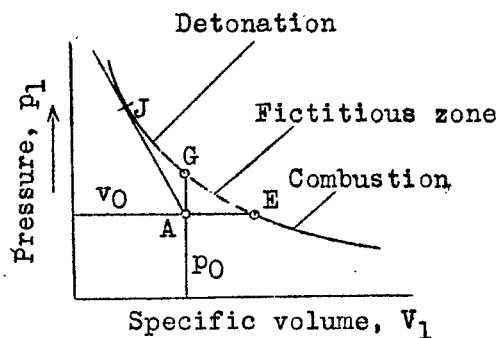


Figure 2.- Hugoniot curve.

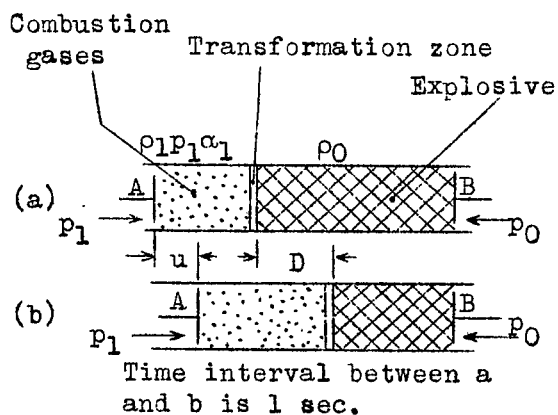


Figure 3.- Improved detonation process.

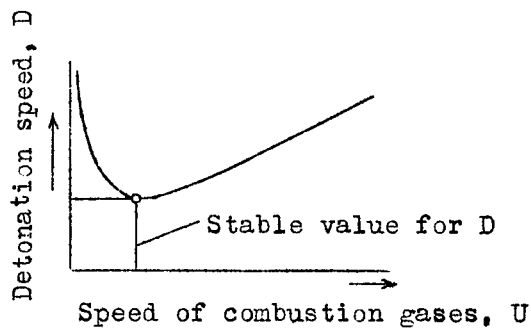
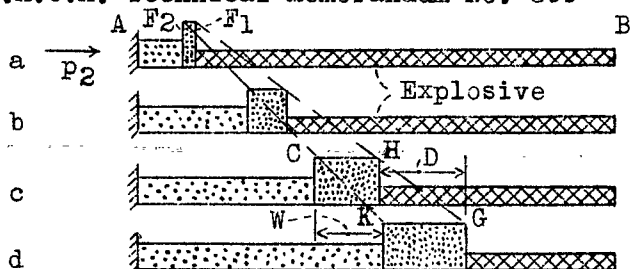


Figure 4.- Detonation speed plotted against speed of combustion gases.



F_1 = Detonation front
 F_2 = Rarefaction front

Figure 5.- Formation of detonation head.

$P_2, \rho_2, \alpha_2 \quad P_1, \rho_1, \alpha_1 \quad P_0, \rho_0$
 Difference in stages = 1 time unit

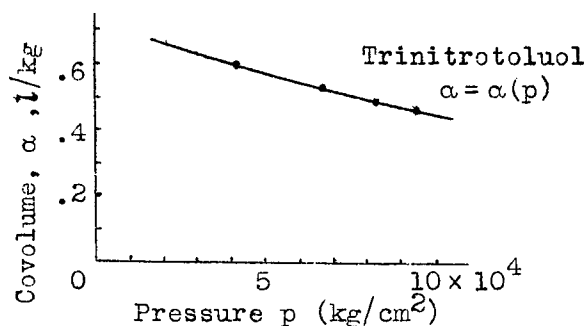


Figure 6.- Covolume of trinitrotoluene plotted against pressure.

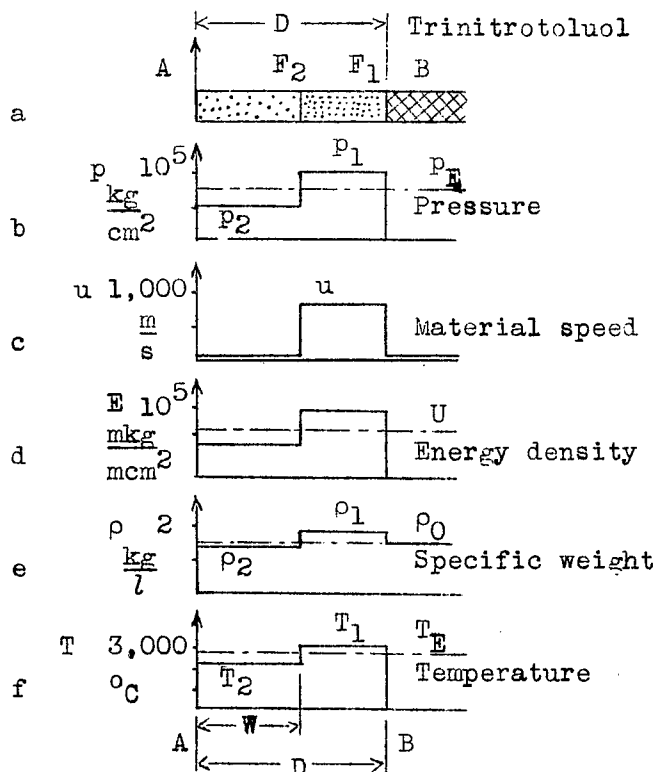


Figure 7.- Distribution of pressure, material speed, energy density, and temperature in a detonating column of trinitrotoluene of D length.

NASA Technical Library



3 1176 01440 6699